# Package 'bvpa'

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**Type** Package

**Title** Bivariate Pareto Distribution

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**Maintainer** Biplab Paul <paul.biplab497@gmail.com>

**Description** Implements the EM algorithm with one-step Gradient Descent method to estimate the parameters of the Block-Basu bivariate Pareto distribution with location and scale. We also found parametric bootstrap and asymptotic confidence intervals based on the observed Fisher information of scale and shape parameters, and exact confidence intervals for location parameters. Details are in Biplab Paul and Arabin Kumar Dey (2023) <doi:10.48550/arXiv.1608.02199> ``An EM algorithm for absolutely continuous Marshall-Olkin bivariate Pareto distribution with location and scale"; E L Lehmann and George Casella (1998) <doi:10.1007/b98854> ``Theory of Point Estimation"; Bradley Efron and R J Tibshirani (1994) <doi:10.1201/9780429246593> ``An Introduction to the Bootstrap"; A P Dempster, N M Laird and D B Rubin (1977) <www.jstor.org/stable/2984875> ``Maximum Likelihood from Incomplete Data via the EM Algorithm".

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bvpa-package *Bivariate Pareto Distribution*

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## Description

Implements the EM algorithm with one-step Gradient Descent method to estimate the parameters of the Block-Basu bivariate Pareto distribution with location and scale. We also found parametric bootstrap and asymptotic confidence intervals based on the observed Fisher information scale, shape parameters, and exact confidence intervals for location parameters.

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## References

Biplab Paul and Arabin Kumar Dey (2023). An EM algorithm for absolutely continuous Marshall-Olkin bivariate Pareto distribution with location and scale, Preprint.

E L Lehmann and George Casella (1998). Theory of Point Estimation, Springer, New York, doi.org/10.1007/b98854.

Bradley Efron and R J Tibshirani (1994). An Introduction to the Bootstrap, CRC press, New York, doi.org/10.1201/9780429246593.

A P Dempster, N M Laird and D B Rubin (1977). Maximum Likelihood from Incomplete Data via the EM Algorithm, Journal of the royal statistical society: series B (methodological), www.jstor.org/stable/2984875.

`conf.intv` *Observed Fisher information based confidence interval of Block-Basu Bivariate Pareto (BBBVPA) distribution*

## Description

Observed Fisher information based confidence interval of Bivariate BBBVPA distribution.

## Usage

```
conf.intv(
  object,
  conf.lev = 0.95,
  tol = 1e-04,
  intv.m1 = c(0, 2),
  intv.m2 = c(0, 2)
)
```

## Arguments

| | |
|---|---|
| `object` | `"bbbvpa"` class object. |
| `conf.lev` | confidence level, $0.95$ (default). |
| `tol` | convergence tolerance for confidence intervals, `0.0001` (default). |
| `intv.m1` | interval related to confidence interval of $\mu_1$, `c(0,2)` (default). |
| `intv.m2` | interval related to confidence interval of $\mu_1$, `c(0,2)` (default). |

## Value

A matrix of lower and upper confidence interval limits (in the first and second column respectively). The matrix rows are labeled by the parameter names (if any) and columns by the corresponding distribution quantiles.

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
# see the example of estimation
```

## conf.intv3 *Observed Fisher information based confidence interval of 3-parameter Block-Basu Bivariate Pareto (BBBVPA) distribution*

### Description

Observed Fisher information based confidence interval of 3-parameter BBBVPA distribution.

### Usage

```
conf.intv3(object, conf.lev = 0.95, tol = 1e-04)
```

### Arguments

| | |
|---|---|
| object | "bbbvpa" class object. |
| conf.lev | confidence level, 0.95 (default). |
| tol | convergence tolerance for confidence intervals, 0.0001 (default). |

### Value

A matrix of lower and upper confidence interval limits (in the first and second column respectively). The matrix rows are labeled by the parameter names (if any) and columns by the corresponding distribution quantiles.

### Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

### Examples

```
dat <- rbb.bvpa(500, 0, 0, 1.0, 1.0, 2.0, 0.4, 0.5)
conf.intv3(estimates3(dat, 2.4, 0.3, 0.6))
```

## estimates *Estimation of Block-Basu Bivariate Pareto (BBBVPA) distribution*

### Description

Parameters estimation of BBBVPA distribution.

## Usage

```
estimates(
  I,
  s1.int,
  s2.int,
  a0.int,
  a1.int,
  a2.int,
  tol.est = 1e-05,
  MxIter.no = 2000,
  rate = 1e-04,
  condition = "log.L"
)
```

## Arguments

| | |
|---|---|
| `I` | bivariate observations. |
| `s1.int` | initial choice of $\sigma_1$. |
| `s2.int` | initial choice of $\sigma_2$. |
| `a0.int` | initial choice of $\alpha_0$. |
| `a1.int` | initial choice of $\alpha_1$. |
| `a2.int` | initial choice of $\alpha_2$. |
| `tol.est` | convergence tolerance, `0.00001` (default). |
| `MxIter.no` | maximum number of iterations, `2000` (default). |
| `rate` | step size or learning rate for gradient descent, `0.0001` (default). |
| `condition` | convergence criterion, `"log.L"` (default) and `"p.logL"`. |

## Value

object of class "bbbvpa", a list consisting of

| | |
|---|---|
| `mu1, mu2, sigma1, sigma2, alpha0, alpha1, alpha2, iter.no` | estimates of parameters and number of iteration. |
| `data` | the supplied data `I`. |

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
# Read data
data(precipitation)
data <- as.vector(precipitation[,2])
data[is.na(data)]<-0
n <- length(data)
```

```
# Construct the three-dimensional data set
data3d <- function(data){
 u <- 12
 Y <- c()
 indx <- indx1 <- indx2 <- indx3 <- 0
 r <- 5
 i <- 2
 while(i < n){
   i <- i + 1
   if(data[i] > u || sum(data[(i-1):i]) > u || sum(data[(i-2):i]) > u){
     if(data[i] > u){imax <- i}
    if(sum(data[(i-1):i]) > u){imax <- i - 3 + which(data[(i-1):i] == max(data[(i-1):i]))[1]}
    if(sum(data[(i-2):i]) > u){imax <- i - 3 + which(data[(i-2):i] == max(data[(i-2):i]))[1]}
     if(max(indx) > (imax-r)){
       cluster <- data[(max(indx)+3):(imax+r)]
     } else{
       cluster <- data[(imax-r):(imax+r)]
     }
     cluster2 <- sapply(c(1:(length(cluster)-1)), function(j) sum(cluster[j:(j+1)]))
     cluster3 <- sapply(c(1:(length(cluster)-2)), function(j) sum(cluster[j:(j+2)]))
     indx1 <- append(indx1,imax-r-1+which(cluster==max(cluster))[1])
     indx2 <- append(indx2,imax-r-1+which(cluster2==max(cluster2)))
     indx3 <- append(indx3,imax-r-1+which(cluster3==max(cluster3)))
     Y <- rbind(Y, c(max(cluster),max(cluster2),max(cluster3)))
     indx <- append(indx,imax)
     i <- i + r
   }
 }
 return(Y)
}
I <- data3d(data)[,c(1,3)]
iniz <- intliz(I)
iniz
est <- estimates(I, iniz[1], iniz[2], iniz[3], iniz[4], iniz[5])
est[-9]
param.boot(I, iniz[1], iniz[2], iniz[3], iniz[4], iniz[5])
conf.intv(est)
```

---

`estimates3` *Estimation of 3-parameter Block-Basu Bivariate Pareto (BBBVPA) distribution*

---

## Description

Parameters estimation of 3-parameter BBBVPA distribution.

## Usage

```
estimates3(
  I,
  a0.int,
  a1.int,
  a2.int,
  tol.est = 1e-05,
  MxIter.no = 2000,
  condition = "log.L"
)
```

## Arguments

| | |
|---|---|
| `I` | bivariate observations. |
| `a0.int` | initial choice of $\alpha_0$. |
| `a1.int` | initial choice of $\alpha_1$. |
| `a2.int` | initial choice of $\alpha_2$. |
| `tol.est` | convergence tolerance, `0.0001` (default). |
| `MxIter.no` | maximum number of iterations, `2000` (default). |
| `condition` | convergence criterion, `"log.L"` (default) and `"p.logL"`. |

## Value

Object of class `"bbbvpa3"`, a list consisting of

| | |
|---|---|
| `alpha0, alpha1, alpha2, iter.no` | estimates of parameters and number of iteration. |
| `data` | the supplied data `I`. |

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
dat <- rbb.bvpa(500, 0, 0, 1.0, 1.0, 2.0, 0.4, 0.5)
estimates3(dat, 2.4, 0.3, 0.6)[-5]
```

intliz *Initialization of Block-Basu Bivariate Pareto (BBBVPA) distribution*

## Description

Return initial choice parameters of BBBVPA distribution.

## Usage

```
intliz(
  data,
  ini.run = 100,
  tol.ini = 0.001,
  proc = "ML",
  intv.s1 = c(0, 5),
  intv.s2 = c(0, 5),
  intv.a0 = c(0, 5),
  intv.a1 = c(0, 5),
  intv.a2 = c(0, 5),
  ...
)
```

## Arguments

| | |
|---|---|
| data | bivariate observations. |
| ini.run | number of random initializations. |
| tol.ini | convergence tolerance, 0.001 (default).. |
| proc | different procedures, "ML" (default) and "S.EM". |
| intv.s1 | interval for random initialization of $\sigma_1$. |
| intv.s2 | interval for random initialization of $\sigma_2$. |
| intv.a0 | interval for random initialization of $\alpha_0$. |
| intv.a1 | interval for random initialization of $\alpha_1$. |
| intv.a2 | interval for random initialization of $\alpha_2$. |
| ... | further arguments to pass to estimates. |

## Value

numeric vector.

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
# see the example of estimation
```

`intliz3` *Initialization of 3-parameter Block-Basu Bivariate Pareto (BBBVPA) distribution*

## Description

Return initial choice parameters of 3-parameter BBBVPA distribution.

## Usage

```
intliz3(
  data,
  ini.run = 100,
  tol.ini = 0.001,
  proc = "ML",
  intv.a0 = c(0, 5),
  intv.a1 = c(0, 5),
  intv.a2 = c(0, 5),
  ...
)
```

## Arguments

| | |
|---|---|
| `data` | bivariate observations. |
| `ini.run` | number of random initializations. |
| `tol.ini` | convergence tolerance, `0.001` (default).. |
| `proc` | different procedures, `"ML"` (default) and `"S.EM"`. |
| `intv.a0` | interval for random initialization of $\alpha_0$. |
| `intv.a1` | interval for random initialization of $\alpha_1$. |
| `intv.a2` | interval for random initialization of $\alpha_2$. |
| `...` | further arguments to pass to `estimates3`. |

## Value

numeric vector.

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
dat <- rbb.bvpa(500, 0, 0, 1.0, 1.0, 2.0, 0.4, 0.5)
intliz3(dat)
```

`logL` *Log-likelihood function of Block-Basu Bivariate Pareto (BBBVPA) distribution*

## Description

Return the log likelihood value.

## Usage

```
logL(I, mu1, mu2, s1, s2, a0, a1, a2)
```

## Arguments

| | |
|---|---|
| `I` | baivariate observations. |
| `mu1` | value of $\mu_1$. |
| `mu2` | value of $\mu_2$. |
| `s1` | value of $\sigma_1$. |
| `s2` | value of $\sigma_2$. |
| `a0` | value of $\alpha_0$. |
| `a1` | value of $\alpha_1$. |
| `a2` | value of $\alpha_2$. |

## Value

a list consisting of

| | |
|---|---|
| `logLik` | A scalar numeric, log likelihood of the model. |
| `n1, n2` | $n_1$ and $n_2$. |

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
dat <- rbb.bvpa(500, 0.1, 0.1, 0.8, 0.8, 2.0, 0.4, 0.5)
logL(dat, 0.1, 0.1, 0.8, 0.8, 2.0, 0.4, 0.5)
```

mLf1 *Marginal log-liklihood function of variable X1*

## Description

Return the marginal log-liklihood value of variable $X_1$.

## Usage

```
mLf1(I, mu1, s1, a0, a1, a2)
```

## Arguments

| | |
|---|---|
| I | baivariate observations. |
| mu1 | value of $\mu_1$. |
| s1 | value of $\sigma_1$. |
| a0 | value of $\alpha_0$. |
| a1 | value of $\alpha_1$. |
| a2 | value of $\alpha_2$. |

## Value

A scalar numeric, the marginal log-liklihood value of variable $X_1$.

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
dat <- rbb.bvpa(500, 0.1, 0.1, 0.8, 0.8, 2.0, 0.4, 0.5)
mLf1(dat, 0.1, 0.8, 2.0, 0.4, 0.5)
```

mLf2 *Marginal log-liklihood function of variable X2*

## Description

Return the marginal log-liklihood value of variable $X_2$.

## Usage

```
mLf2(I, mu2, s2, a0, a1, a2)
```

### Arguments

| | |
|---|---|
| I | baivariate observations. |
| mu2 | value of $\mu_2$. |
| s2 | value of $\sigma_2$. |
| a0 | value of $\alpha_0$. |
| a1 | value of $\alpha_1$. |
| a2 | value of $\alpha_2$. |

### Value

A scalar numeric, the marginal log-liklihood value of variable $X_2$.

### Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

### Examples

```
dat <- rbb.bvpa(500, 0.1, 0.1, 0.8, 0.8, 2.0, 0.4, 0.5)
mLf2(dat, 0.1, 0.8, 2.0, 0.4, 0.5)
```

---

## param.boot *Parametric bootstrap confidence intervals of parameters of Block-Basu Bivariate Pareto (BBBVPA) distribution*

---

### Description

Parametric bootstrap confidence interval of parameters of BBBVPA distribution.

### Usage

```
param.boot(
  data,
  s1.int,
  s2.int,
  a0.int,
  a1.int,
  a2.int,
  conf.lev = 0.95,
  intv.m1 = c(0, 2),
  intv.m2 = c(0, 2),
  no.paboot = 100,
  tol = 1e-04,
  ...
)
```

### Arguments

| | |
|---|---|
| `data` | bivariate observations. |
| `s1.int` | initial choice of $\sigma_1$. |
| `s2.int` | initial choice of $\sigma_2$. |
| `a0.int` | initial choice of $\alpha_0$. |
| `a1.int` | initial choice of $\alpha_1$. |
| `a2.int` | initial choice of $\alpha_2$. |
| `conf.lev` | confidence level, defult $0.95$. |
| `intv.m1` | interval related to confidence interval of $\mu_1$, `c(0,2)` (default). |
| `intv.m2` | interval related to confidence interval of $\mu_1$, `c(0,2)` (default). |
| `no.paboot` | number of bootstrap samples, `100` (default). |
| `tol` | convergence tolerance for confidence interval of $\mu_1$. and $\mu_2$, `0.0001` (default). |
| `...` | further arguments to pass to `estimates`. |

### Value

A matrix of lower and upper confidence interval limits (in the first and second column respectively). The matrix rows are labeled by the parameter names (if any) and columns by the corresponding distribution quantiles.

### Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

### Examples

```
# see the example of estimation
```

---

## `param.boot3` *Parametric bootstrap confidence intervals of parameters of 3-parameter Block-Basu Bivariate Pareto (BBBVPA) distribution*

---

### Description

Parametric bootstrap confidence interval of parameters of 3-parameter BBBVPA distribution.

### Usage

```
param.boot3(
  data,
  a0.int,
  a1.int,
  a2.int,
  conf.lev = 0.95,
  no.paboot = 100,
  ...
)
```

### Arguments

| | |
|---|---|
| data | bivariate observations. |
| a0.int | initial choice of $\alpha_0$. |
| a1.int | initial choice of $\alpha_1$. |
| a2.int | initial choice of $\alpha_2$. |
| conf.lev | confidence level, defult 0.95. |
| no.paboot | number of bootstrap samples, 100 (default). |
| ... | further arguments to pass to estimates3. |

### Value

A matrix of lower and upper confidence interval limits (in the first and second column respectively). The matrix rows are labeled by the parameter names (if any) and columns by the corresponding distribution quantiles.

### Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

### Examples

```
dat <- rbb.bvpa(500, 0, 0, 1.0, 1.0, 2.0, 0.4, 0.5)
param.boot3(dat, 2.4, 0.3, 0.6)
```

---

## pctl.fun *Survival functions of pivots of estimators of locations.*

---

### Description

Survival functions of pivots of estimators of locations $\mu_1$ and $\mu_2$. These are required to calculate the critical value of confidence intervals for $\mu_1$ and $\mu_2$.

### Usage

```
pctl.fun(z, n, a0, a1, a2, pct, select = 1)
```

### Arguments

| | |
|---|---|
| z | quantiles. |
| n | number of observations. |
| a0 | value of $\alpha_0$. |
| a1 | value of $\alpha_1$. |
| a2 | value of $\alpha_2$. |
| pct | probabilities. |
| select | Allows to select the function for different location parameters. a single model term to be selected for printing. e.g. if you just want the function for $\mu_1$ set $select = 1$ (default). |

### Value

return a function.

### Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

### Examples

```
uniroot(pctl.fun, interval=c(0,2), n = 500, a0 = 2.0, a1 = 0.4, a2 = 0.5,
 pct = 0.025, tol = 0.0001)[[1]]
```

---

## precipitation *Precipitation data*

---

### Description

The dataset contains daily accumulated precipitation data (in mm) from Abisko Scientific Research Station in northern Sweden for 100 years, from 1st January 1913 to 31st December 2012.

### Usage

```
data(precipitation)
```

### Format

A data frame with 36524 rows and 2 columns and the following variables:

**Time** 1st column represents Day.

**Precipitation..mm.** 2nd column represents daily accumulated precipitation (in mm) of the day.

## Source

<https://www.polar.se/stoed-till-polarforskning/abisko-naturvetenskapliga-station/>

## Examples

```
data(precipitation)
```

---

`pseu.logL` *Pseudo log-likelihood function of Block-Basu Bivariate Pareto (BBB-VPA) distribution*

---

## Description

Return the pseudo log likelihood value.

## Usage

```
pseu.logL(I, mu1, mu2, s1, s2, a0, a1, a2)
```

## Arguments

| | |
|---|---|
| `I` | baivariate observations. |
| `mu1` | value of $\mu_1$. |
| `mu2` | value of $\mu_2$. |
| `s1` | value of $\sigma_1$. |
| `s2` | value of $\sigma_2$. |
| `a0` | value of $\alpha_0$. |
| `a1` | value of $\alpha_1$. |
| `a2` | value of $\alpha_2$. |

## Value

A scalar numeric, pseudo log likelihood of the model.

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
dat <- rbb.bvpa(500, 0.1, 0.1, 0.8, 0.8, 2.0, 0.4, 0.5)
pseu.logL(dat, 0.1, 0.1, 0.8, 0.8, 2.0, 0.4, 0.5)
```

rbb.bvpa *Simulate from a Block-Basu Bivariate Pareto (BBBVPA) distribution*

## Description

Produces one or more samples from the specified BBBVPA distribution.

## Usage

```
rbb.bvpa(n, mu1, mu2, sig1, sig2, alp0, alp1, alp2)
```

## Arguments

| | |
|---|---|
| n | number of observations. |
| mu1 | value of $\mu_1$ |
| mu2 | value of $\mu_2$ |
| sig1 | value of $\sigma_1$ |
| sig2 | value of $\sigma_2$ |
| alp0 | value of $\alpha_0$ |
| alp1 | value of $\alpha_1$ |
| alp2 | value of $\alpha_2$ |

## Value

numeric matrix.

## Author(s)

Biplab Paul <paul.biplab497@gmail.com> and Arabin Kumar Dey <arabin@iitg.ac.in>

## Examples

```
cor(rbb.bvpa(500, 0.1, 0.1, 0.8, 0.8, 2.0, 0.4, 0.5))
```

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